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Model for rumor spreading over networks

Daniel Trpevski
Macedonian Academy for Sciences and Arts, Skopje, Macedonia

Wallace K. S. Tang
Department of Electronic Engineering, City University of Hong Kong, Hong Kong, Hong Kong SAR

Ljupco Kocarev
Macedonian Academy for Sciences and Arts, Skopje, Macedonia and BioCircuits Institute, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0402, USA

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An alternate model for rumor spreading over networks is suggested, in which two rumors (termed rumor 1 and rumor 2) with different probabilities of acceptance may propagate among nodes. The propagation is not symmetric in the sense that when deciding which rumor to adopt, nodes always consider rumor 1 first. The model is a natural generalization of the well-known epidemic SIS (susceptible-infective-susceptible) model and reduces to it when some of the parameters of this model are zero. We find that preferred rumor 1 is dominant in the network when the degree of nodes is high enough and/or when the network contains large clustered groups of nodes, expelling rumor 2. However, numerical simulations on synthetic networks show that it is possible for rumor 2 to occupy a nonzero fraction of the nodes in many cases as well. Specifically, in the Watts-Strogatz small-world model a moderate level of clustering supports its adoption, while increasing randomness reduces it. For Erdos-Renyi networks, a low average degree allows the coexistence of the two types of rumors. In Barabasi-Albert networks generated with a low m, where m is the number of links when a new node is added, it is also possible for rumor 2 to spread over the network.

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I. INTRODUCTION

The investigation of social spreading phenomena such as the propagation of rumors, the diffusion of fads, the adoption of technological innovations, and the success of consumer products mediated by word of mouth, has a long tradition in sociology and economics. Effects of the network of contacts in the spreading process have been postulated long since [1,2], and recently, with the development of the theory of complex networks, these effects are gradually unraveled. In fact, in the last decade, complex networks theory has paved the way for exploring many real-world large-scale networks, and describing and understanding various processes that play out on the nontrivial topology of these networks. Much of the studies in this respect are concerned with spreading processes such as virus propagation in social and computer networks [3–8], the diffusion of innovations [9,10], the occurrence of information cascades in social and economic systems [11,12], disaster spreading in infrastructures [13], or information diffusion in a society through the word-of-mouth (w-o-m) mechanism [14], to name a few.

The most popular model for information or rumor spreading, introduced by Daley and Kendall [15], see also [16–19], is conceptually similar to the SIR (susceptible-infective-recovered) model for epidemiology. Agents are divided into three classes: ignorants, spreaders, and stiflers, i.e., those who have lost interest in diffusing the information or rumor. Their role is exactly the same as the susceptible, infective, and recovered agents of the SIR model, respectively. Epidemiological models have since been repeatedly used for describing information spread, such as, topic flow in blogspace [20], and word of mouth in product marketing [14].

Other widely used models for describing collective social behavior are the threshold models, first proposed by Granovetter in [21]. Each individual has a specific threshold, based on which a binary decision is made. More formal definitions which take social network structure into account have appeared since, the simplest version of which is the linear threshold model [22]. A variant of the threshold model has been used, for example, in [9,10] for describing diffusion of innovations in a population. The effects of network topology for the threshold model have been analyzed by [11,12].

However, in the context of complex networks research, so far the spread of only one type of information through a network has been considered (a notable exception is [23], as we outline later). In this paper we present a model of rumor (information) spreading, where two different types of information affect the nodes, and consider the behavior of the model for different network topologies. We term the two types of information rumor 1 and rumor 2.

The model we propose is a natural extension of the SIS epidemiological model, where a node in the network can be in the *susceptible* state, not having contracted the disease, or in the *infective* state, able to spread the disease to each of its neighbors. Infectives recover, becoming susceptible to the disease again. In our model the nodes are divided into three classes: ignorants, rumor 1 spreaders, and rumor 2 spreaders. Namely, each node is "susceptible" to the effects of two kinds of "infections" and is able to recover from them as well. Moreover, the source of rumor 1 has a dominant position in the sense that the nodes will always consider rumors coming from this node first. This formulation could be significant for describing two w-o-m processes circulating in a social network, e.g., concerning two competing products in the market, or two candidates form opposing parties in elections, one of which has a better standing in the public eye. We note that rumor spreading also has appealing connections with the search for robust scalable communication protocols in large distributed systems [24,25], "viral" strategies in marketing [26], and epidemic routing in *ad hoc* networks [27,28].

When considering rumor spreading, some of the relevant questions are similar to those for epidemiology: How many nodes will eventually be reached by the rumor? Is there an "epidemic threshold" (i.e., critical point) for the rate of spreading, separating a regime in which a finite fraction of nodes will be informed from one where the rumor remains confined to a small neighborhood? There is a large amount of studies concerning epidemic spreading from a complex networks perspective. Using percolation theory ideas and generating function methods [3] give exact analytical results for the epidemic threshold, outbreak size, and other relevant quantities for the SIR model. The results represent average values over an ensemble of random graphs with an arbitrary degree distribution. In [4,5], the absence of an epidemic threshold has been established for the SIS model and infinitely large networks with a power-law degree distribution $k^{-\gamma}$, $2<\gamma\leq 3$, whereas [6] shows the existence of a threshold for clustered power-law networks. Rather than determining the epidemic threshold for a whole class of networks with a given degree distribution, [7,8] propose its calculation for a specific network given with an adjacency matrix, for a SIR and SIS model, respectively. We follow this idea to determine the rumor spreading threshold in our model. As to the effect of network topology on the spreading process, Watts-Strogatz small-world networks have been found to be less susceptible to spreading processes as network randomness increases in threshold models [12], and more susceptible with increasing randomness for the SIR model [18,19]. Power-law networks are reported as more resilient to large outbreaks than comparable uniform random graphs for threshold spreading models with random initial spreaders [11], and less resilient for the SIS model [4].

Finally, we outline a study which is close to our work in the sense that two rumor types spread through the network. Namely, in [23], Goldenberg *et al.* investigated the effects of both positive and negative w-o-m on a firm's profits. In this study, negative w-o-m is limited to traveling two hops away from its source. In our model, both rumor types can spread arbitrarily far from their respective sources.

The paper proceeds as follows. Section II defines the model and analyzes the stability of its dynamics. In Sec. III we describe the behavior of the model on regular network topologies, namely the star and fully connected topology. Results of the rumor dissemination simulations on complex network topologies are given in Sec. IV. The last section concludes the paper and points out potential research directions.

II. RUMOR SPREADING MODEL

A. Definition of the model

Consider a closed population of N individuals, connected by a neighborhood structure which is represented by an undirected unweighted graph $G=(V,E)$ with node set V and edge set E. Let A denote the adjacency matrix of the graph G, i.e., $a_{ij}=1$ if $(i,j)\in E$ and $a_{ij}=0$ otherwise. We propose a discrete stochastic model for rumor spreading in a network. The model allows for two different types of rumor coming from two different sources, to spread through the network. At time k, each node i can be in one of three possible states: 1, 2, and 3. The state of the node is indicated by a status vector, an indicator vector containing a single 1 in the position corresponding to the present state, and 0 everywhere else,

$$\mathbf{s}_i(k)=[s_i^1(k)s_i^2(k)s_i^3(k)]^T,$$

for all $i\in 1,\ldots,N$. A node being in state 1 or 2 indicates that it is a supporter, or adopter of rumor 1 or 2, accordingly. State 3 signifies an undecided or neutral state of the node in relation to the two rumors circulating in the network. Let

$$\mathbf{p}_i(k)=[p_i^1(k)p_i^2(k)p_i^3(k)]^T$$

be the probability mass function of node i at time k. For every node i it states the probability of being in each of the possible states at time k. Having defined that, the equations describing the dynamics of each node, i.e., the evolution of the model, are

$$p_i^1(k+1)=s_i^3(k)f_i+a_1s_i^1(k)$$

$$p_i^2(k+1)=s_i^3(k)(1-f_i)g_i+a_2s_i^2(k)$$

$$p_i^3(k+1)=s_i^3(k)(1-f_i)(1-g_i)+(1-a_1)s_i^1(k)+(1-a_2)s_i^2(k)$$

$$\mathbf{s}_i^T(k+1)=MultiRealize[\mathbf{p}_i^T(k+1)] \tag{1}$$

where $MultiRealize[\cdot]$ performs a random realization for the probability distribution given with $\mathbf{p}_i^T(k+1)$. a_1 and a_2 are parameters, $0\leq a_1\leq 1$ and $0\leq a_2\leq 1$. In the model f_i and g_i are given by

$$f_i(k)=1-\prod_{j=1}^{N}[1-\beta a_{ij}s_j^1(k)],$$

$$g_i(k)=1-\prod_{j=1}^{N}[1-\gamma a_{ij}s_j^2(k)]. \tag{2}$$

In the last two expressions β and γ are parameters, $0\leq\beta\leq 1$ and $0\leq\gamma\leq 1$. We note that a discrete stochastic SIS model can be obtained as a special case of the rumor spreading model by setting $a_2=0$, $\gamma=0$ and no nodes in status 2 initially. Status 1 is then the infective state, and status 3 is the susceptible state, with the curing rate of the disease being $\delta=1-a_1$.

The mechanism of spreading is the following. Each node in status 1 attempts to send a message to each of its neighbors at the beginning of time k. Each attempt is successful with probability β and is independent of other attempts. The nodes in status 2 also send messages to each of their neighbors with probability γ. Hence, f_i and g_i are the probabilities that node i receives a status 1 or status 2 message from its neighboring supporters of rumors 1 and 2, accordingly. However, note from the formulation of Eq. (1) that node i will actually be able to become a supporter of rumor 2 at $(k+1)$ only if, at time k, it does not receive a status 1 message from its neighbors. More precisely, the probability of converting from an undecided status to status 2 is not g_i, but it is multiplied by the factor $(1-f_i)$ which in general is smaller than 1. Conversely, node i can become a supporter of rumor 1 regardless of whether it receives a status 2 message or not. In this way, the model has prebuilt a preference in each node for the rumor of type 1, as if though its source was more credible or more reputable. Additionally, after adopting a particular type of rumor, the nodes in states 1 and 2 continue to preserve their status at a rate of a_1 and a_2, respectively, i.e., they convert back to status 3 with a rate of $(1-a_1)$ and $(1-a_2)$, respectively. The parameters a_1 and a_2 can be said to signify the remembrance rate of each rumor, or how long the nodes are willing to support the adopted rumor before abandoning it and converting back to an undecided status.

A model of this kind could potentially be useful for a variety of real-world situations. In marketing circumstances, one can imagine a product entering the market and having to compete with a product of the same kind from an already established brand-name company. In a situation of elections, one could take the case of a country which has a dominant political party, and each citizen is inclined to consider a candidate from this party first. However, since in the model rumors spread through the interaction of nodes, it is safe to use it for simulating the spread of w-o-m about two different products, or two candidates from opposing parties. The model does not encompass factors such as mass-media marketing, or political campaigns, which have an effect on all individuals in a population. Furthermore, the model makes a simplifying assumption that the remembrance rates a_1 and a_2 and the message transmission probabilities β and γ are the same for all nodes. In a real-world scenario where each node represents an individual, they would depend on one's characteristics and preferences.

Let $X(k)=\Sigma_{i=1}^{N}s_i^1(k)$, $Y(k)=\Sigma_{i=1}^{N}s_i^2(k)$, and $Z(k)=\Sigma_{i=1}^{N}s_i^3(k)$ be the total number of nodes in statuses 1, 2, and 3 at time k, respectively. Further, let $N_1=\mathbb{E}[X(\infty)]$, $N_2=\mathbb{E}[Y(\infty)]$, and $N_3=\mathbb{E}[Z(\infty)]$. The object of interest is the average number of nodes that eventually (when $k\rightarrow\infty$) adopt statuses 1 and 2, N_1 and N_2, compared to the total number of nodes N in the network.

In the following, in order to facilitate the mathematical analysis we rewrite the model given with Eqs. (1) and (2) as

$$\begin{aligned} p_i^1(k+1) &= p_i^3(k)f_i + a_1 p_i^1(k) \\ p_i^2(k+1) &= p_i^3(k)(1-f_i)g_i + a_2 p_i^2(k) \\ p_i^3(k+1) &= p_i^3(k)(1-f_i)(1-g_i) + (1-a_1)p_i^1(k) \\ &\quad + (1-a_2)p_i^2(k) \end{aligned} \tag{3}$$

and f_i and g_i as

$$f_i(k) = 1 - \prod_{j=1}^{N}[1-\beta a_{ij}p_j^1(k)]$$

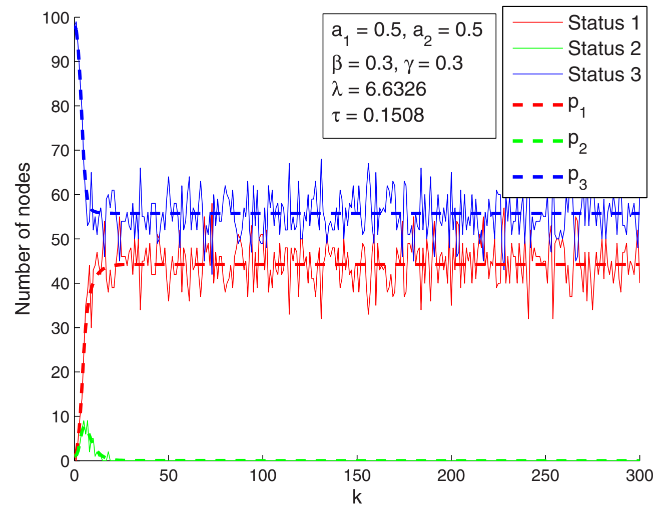


FIG. 1. (Color online) Evolution of Eqs. (1) and (3) on a 100-node BA network generated with $m=2$. Initially, node 25 is a supporter of rumor 1, while node 29 is of rumor 2. Both nodes have degree 2 and may change their status as time progresses. The solid lines show the number of nodes in each status as simulated by Eq. (1), while the dashed lines show the evolution of the average number of nodes in each status, i.e., the evolution of $\Sigma_{i=1}^{N}p_i^1(k)$, $\Sigma_{i=1}^{N}p_i^2(k)$, and $\Sigma_{i=1}^{N}p_i^3(k)$ in Eq. (3).

$$g_i(k) = 1 - \prod_{j=1}^{N}[1-\gamma a_{ij}p_j^2(k)] \tag{4}$$

Equivalently N_1,N_2,N_3 can be computed using Eq. (3) as $N_1=\Sigma_{i=1}^{N}p_i^1(\infty)$, $N_2=\Sigma_{i=1}^{N}p_i^2(\infty)$, and $N_3=\Sigma_{i=1}^{N}p_i^3(\infty)$.

To illustrate that Eqs. (1) and (3) are the same, consider Fig. 1. The figure shows the results of running both of the models on a 100 node scale-free network generated by the Barabasi-Albert (BA) algorithm as given in [29], with $m=2$. In this case we simulate injecting the two rumors at two sources initially, and the sources are allowed to change their status as time passes. As can be seen, the evolution of the sum probability vector according to Eq. (3), corresponds to the evolution of the number of nodes in each status as predicted by Eq. (1). In effect, by studying Eq. (3), we are investigating the evolution of the probability vectors from which random realizations are made in Eq. (1).

B. Dynamical systems approach

In this part we apply a dynamical systems approach to our model. Let us in Eq. (3) replace the probabilities for the node i to be in status 1 and 2 with $x_i=p_i^1$ and $y_i=p_i^2$, respectively. The evolution of the model can be rewritten as

$$\begin{aligned} x_i(k+1) &= [1-x_i(k)-y_i(k)]f_i(k) + a_1 x_i(k) \\ y_i(k+1) &= [1-x_i(k)-y_i(k)][1-f_i(k)]g_i(k) + a_2 y_i(k), \end{aligned} \tag{5}$$

where

$$f_i(k) = 1 - \prod_{j=1}^{N}[1-\beta a_{ij}x_j(k)]$$

$$g_i(k) = 1 - \prod_{j=1}^{N} [1 - \gamma a_{ij} y_j(k)]. \tag{6}$$

Equation (5) represents a nonlinear dynamical system $F:[0,1]^{2N} \to [0,1]^{2N}$. Since x_i and y_i are probabilities, and assuming that the corresponding graph is connected, then the ergodicity of the Markov chain of the whole system is guaranteed [30] if

$$a_1 \neq 1, \ a_2 \neq 1, \ \beta \neq 0, \ \beta \neq 1, \ \gamma \neq 0, \ \gamma \neq 1. \tag{7}$$

Therefore, when condition (7) is satisfied, dynamical system (5) has a unique globally stable fixed point.

System (5) has a fixed point at $(x_i, y_i) = (0,0)$ for all i. The local stability of this fixed point can be analyzed using the Jacobian matrix of system (5) evaluated at the fixed point,

$$DF|_{(0,0)} = \begin{bmatrix} A_\beta & 0_N \\ 0_N & A_\gamma \end{bmatrix},$$

where $A_\beta = a_1 I + \beta A$, $A_\gamma = a_2 I + \gamma A$, and 0_N (the matrix of all 0 elements) are $N \times N$ matrices. Hence the fixed point (0,0) is stable when

$$\max\{a_1 + \beta\lambda, a_2 + \gamma\lambda\} < 1, \tag{8}$$

where λ is the largest eigenvalue of the adjacency matrix. What this condition means with regard to rumor spread is that, whenever it is fulfilled, eventually no rumor of either type will persist in the network. All of the nodes will be in the neutral status, since the model stabilizes in a state where the probabilities of each node to adopt either status 1 or 2 are zero.

Restating condition (8) as

$$\frac{\beta}{1 - a_1} < \frac{1}{\lambda}, \frac{\gamma}{1 - a_2} < \frac{1}{\lambda}, \tag{9}$$

one can see that the value of $\tau = \frac{1}{\lambda}$ appears as a threshold value for the ratios $\frac{\beta}{1-a_1}$ and $\frac{\gamma}{1-a_2}$ up to which the fixed point (0,0) is stable. In these ratios, β and γ are the message transmission probabilities. Since we interpreted a_1 and a_2 as the remembrance rates of the two rumors, $(1-a_1)$ and $(1-a_2)$ are the rates at which the rumors dissipate at the nodes. Whenever a ratio of transmission to dissipation is larger than the threshold, the fixed point $(x_i, y_i) = (0,0)$ is unstable, and there will be rumor spreading in the network. For both rumor types to be able to spread in a network, it is necessary that both transmission to dissipation ratios are greater than the network threshold. However, whether at the end both rumors will still persist in the network depends on the particulars of the case, as we discuss later in the paper.

The existence of a network threshold $\frac{1}{\lambda}$ has already been pointed out in several epidemiological models of virus spreading such as the SIS-type model in [7] and the SIR model in [8]. Note that this threshold value is a critical point in the system dynamics, and is not related to node specific thresholds in the class of threshold models.

III. BEHAVIOR OF THE MODEL ON REGULAR NETWORK TOPOLOGIES

This section presents results on the model behavior on regular networks. The topologies considered are the star and fully connected network. In all the experiments initially there is one node in status 1 and one in status 2. The rumors are just injected in these two sources, allowing them to change their status as time passes. Also recall that for the convergence of the model to a unique fixed point we established condition (7). It will hold for all the numerical simulations.

A. Star network

In this section results on the star topology are presented. In all cases, node 1 is the hub to which all the leafs are connected. Because of its central position in the network, one can expect the hub to have a major role as a rumor spreader. The model equations for the star become

$$x_1(k+1) = [1 - x_1(k) - y_1(k)] \left\{ 1 - \prod_{j=2}^{N} [1 - \beta x_j(k)] \right\} + a_1 x_1(k)$$

$$x_i(k+1) = [1 - x_i(k) - y_i(k)] \beta x_1(k) + a_1 x_i(k), \quad i = 2, \ldots, N$$

$$y_1(k+1) = [1 - x_1(k) - y_1(k)] \prod_{j=2}^{N} [1 - \beta x_j(k)] \left\{ 1 - \prod_{j=2}^{N} [1 - \gamma y_j(k)] \right\} + a_2 y_1(k)$$

$$y_i(k+1) = [1 - x_i(k) - y_i(k)][1 - \beta x_1(k)] \gamma y_1(k) + a_2 y_i(k), \quad i = 2, \ldots, N. \tag{10}$$

Figure 2 shows the case when only one of the rumors exceeds the threshold for the star for which $\lambda = \sqrt{N-1}$. Thus, only one status prevails, and all the leafs have the same value for the probability of this status. If we write $x_1 = a$, $x_i = b$, $i = 2, \ldots, N$ and $y_j = 0 \, \forall j$, or $y_1 = a$, $y_i = b$, $i = 2, \ldots, N$, and $x_j = 0$, $\forall j$, depending on which the remaining status is, the fixed point of the star, using Eq. (10), is

$$a = \frac{1 - \left(1 - \frac{\beta^2 a}{\beta a + 1 - a_1}\right)^{N-1}}{1 - \left(1 - \frac{\beta^2 a}{\beta a + 1 - a_1}\right)^{N-1} + 1 - a_1}, \quad b = \frac{\beta a}{\beta a + 1 - a_1} \tag{11}$$

or

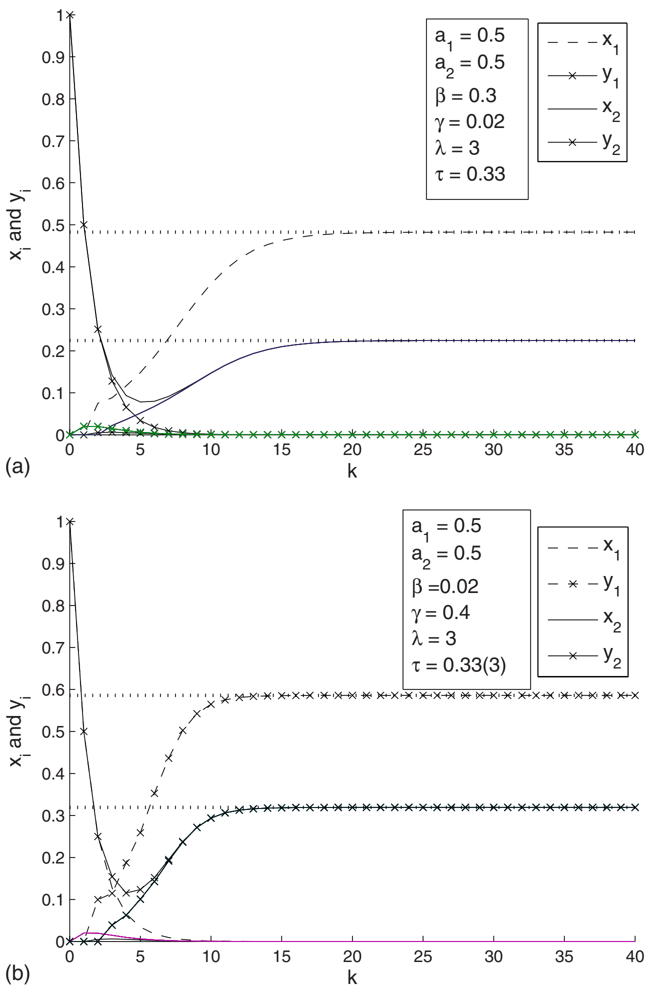


FIG. 2. (Color online) The evolution of x_i (lines) and y_i (crossed lines) for a star with 10 nodes. The dotted lines are the stable values as computed by iterating Eqs. (11) and (12) accordingly. (a) $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} < \frac{1}{\lambda}$. Initially the hub is in status 2, and node 2 in status 1. (b) $\frac{\beta}{1-a_1} < \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$. At $k=0$, the hub is in status 1 and node 2 is in status 2.

$$a = \frac{1-\left(1-\dfrac{\gamma^2 a}{\gamma a + 1 - a_2}\right)^{N-1}}{1-\left(1-\dfrac{\gamma^2 a}{\gamma a + 1 - a_2}\right)^{N-1} + 1 - a_2}, \quad b = \frac{\gamma a}{\gamma a + 1 - a_2}. \tag{12}$$

Iterating these equations for the hub on (0,1] will give the correct stable values, as Fig. 2 shows.

When both rumors can spread in the network, both status 1 and status 2 can appear in the stable state (Fig. 3). Also, due to the regularity of the topology, the leaves have equal x_i and y_i values. Writing $(x_i, y_i) = (x_2, y_2)$, $i = 2, \ldots, N$ in Eq. (10) we get

$$x_1 = z_1[1-(1-\beta x_2)^{N-1}] + a_1 x_1,$$

$$x_2 = z_2 \beta x_1 + a_1 x_2,$$

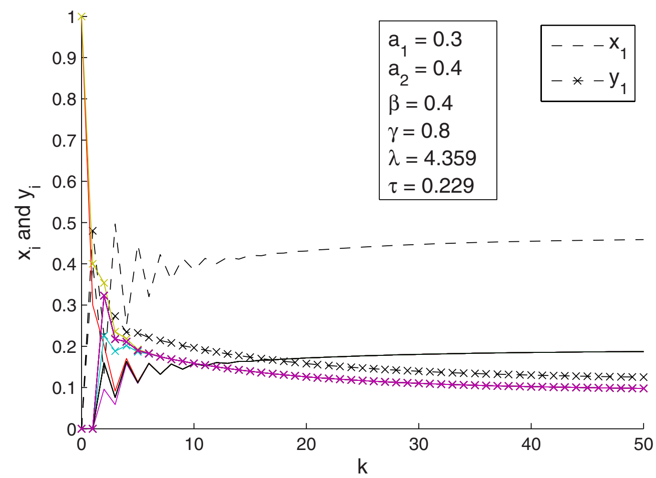


FIG. 3. (Color online) Evolution of Eq. (10) for a star with 20 nodes. Initially, one leaf supports rumor 1, and one leaf has adopted rumor 2. Lines are for x_i and crossed lines are for y_i, $i=1,\ldots,N$.

$$y_1 = z_1(1-\beta x_2)^{N-1}[1-(1-\gamma y_2)^{N-1}] + a_2 y_1,$$

$$y_2 = z_2(1-\beta x_1)\gamma y_1 + a_2 y_2,$$

where $z_1 = 1 - x_1 - y_1$ and $z_2 = 1 - x_2 - y_2$. From the last four equations we can see the behavior of the model in the limit of large N. In fact $(1-\beta x_2)^{N-1} \approx 0$ when N is large, and it follows that $y_i \approx 0$, $\forall i$, while

$$x_1 = \frac{1}{2-a_1}, \quad x_2 = \frac{\beta}{\beta + (1-a_1)(2-a_1)}. \tag{13}$$

This is shown on Fig. 4, for a star with 1000 nodes. In words, in such a situation it is very difficult for rumor 2 to pervade the network. Once the hub hears the word from someone in status 1, it spreads it to all the other nodes in the network, and they will always consider this message first.

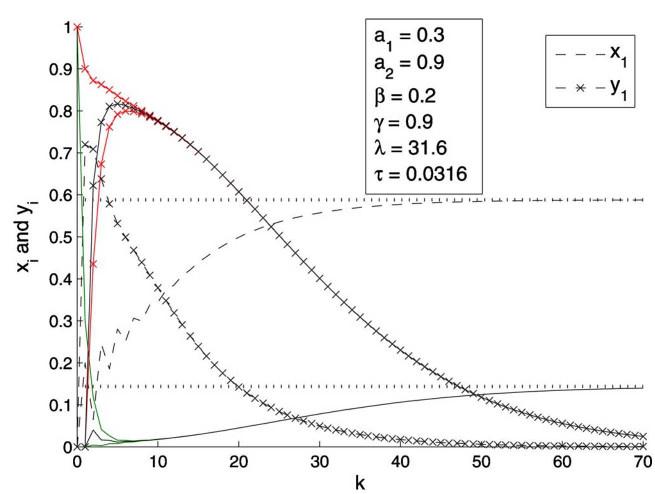


FIG. 4. (Color online) The evolution of (10) for a 1000-node star. Initially, one of the leafs is in status 1, and one is in status 2. Lines are for x_i and crossed lines are for y_i. The dotted lines are approximations for the stable values of the hub and leaves, as given by Eq. (13).

B. Fully connected network

A complete, or fully connected network, is a simple network in which every pair of distinct nodes is connected by an edge. The fully connected network is an example of an environment with homogeneous mixing where every node has equal contact with the others. The model equations for this topology become

$$x_i(k+1) = [1 - x_i(k) - y_i(k)]\left\{1 - \prod_{j=1, j\neq i}^{N} [1 - \beta x_j(k)]\right\} + a_1 x_i(k)$$

$$y_i(k+1) = [1 - x_i(k) - y_i(k)] \prod_{j=1, j\neq i}^{N} [1 - \beta x_j(k)]\left\{1 - \prod_{j=1, j\neq i}^{N} [1 - \gamma y_j(k)]\right\} + a_2 y_i(k). \quad (14)$$

We present several results. Figure 5 shows cases when only one of the rumors has a transmission to dissipation ratio greater than τ. Accordingly, only one type of message is able to spread through the network. The stable values x_i or y_i are the same for all the nodes. Using this in Eq. (14) we obtain

$$\tilde{x} = \frac{1 - (1 - \beta\tilde{x})^{N-1}}{1 - (1 - \beta\tilde{x})^{N-1} + 1 - a_1} \quad (15)$$

or

$$\tilde{y} = \frac{1 - (1 - \gamma\tilde{y})^{N-1}}{1 - (1 - \gamma\tilde{y})^{N-1} + 1 - a_2} \quad (16)$$

for the value of the stable state in Figs. 5(a) and 5(b). Iterating each of these equations on (0,1] instead of Eq. (14) will also give the stable state solution. When N is large enough, the solutions for the one-status-spreading case become independent of the message transmission rates, and are well approximated by

$$\tilde{x} = \frac{1}{2 - a_1}$$

$$\tilde{y} = \frac{1}{2 - a_2}. \quad (17)$$

When both $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$, both message types are able to spread through the network. For the parameter values on Fig. 6, each node has the possibility of being in each of the three statuses, and on average most nodes will be in status 2. Due to the regularity of the topology, all the nodes have the same stable values for all the probabilities. Using this fact, from Eq. (14) one obtains

$$\tilde{x} = (1 - \tilde{x} - \tilde{y})[1 - (1 - \beta\tilde{x})^{N-1}] + a_1\tilde{x}$$

$$\tilde{y} = (1 - \tilde{x} - \tilde{y})(1 - \beta\tilde{x})^{N-1}[1 - (1 - \gamma\tilde{y})^{N-1}] + a_2\tilde{y} \quad (18)$$

for the value of the fixed point. An interesting observation is that when N is large enough, the fixed point becomes

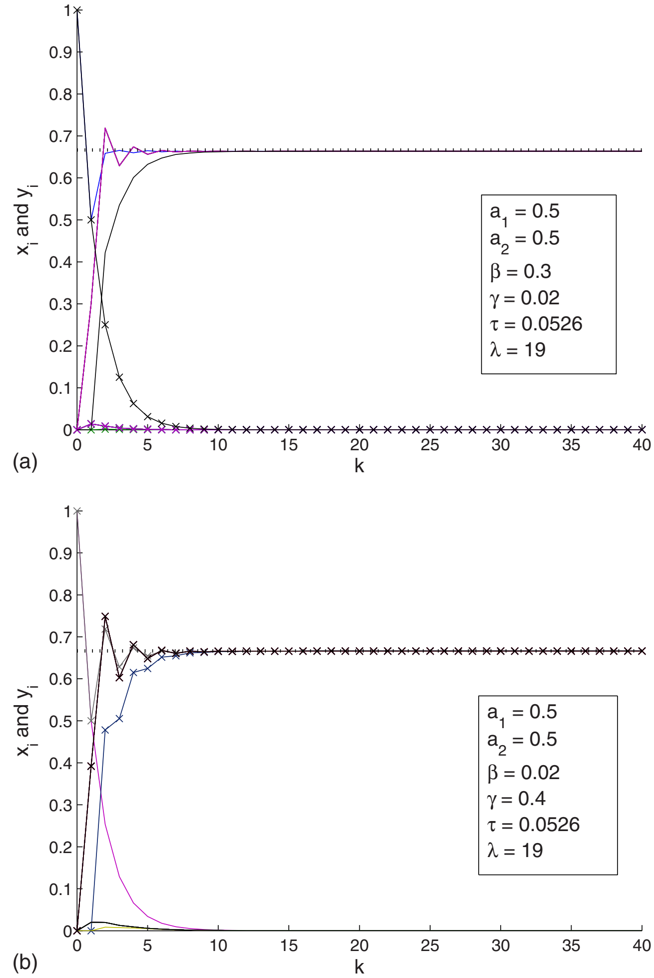


FIG. 5. (Color online) The evolution of x_i (solid lines) and y_i (crossed lines) for a 20-node fully connected network. The dotted lines are the approximated stable values [Eq. (17)]. (a) Only rumor 1 has parameters above the network threshold. (b) Only rumor 2 has parameters above the network threshold.

$$\tilde{x} = \frac{1}{2 - a_1},$$

$$\tilde{y} = 0. \quad (19)$$

An illustration of this is in Fig. 7, for a fully connected network of 100 nodes and one node in each status initially. Although messages of type 2 pervade the network at the beginning due to high a_2 and γ, status 2 is not sustained in the network.

A note on the results for large stars and fully connected graphs is in order. In the limit of large N, the hub and the fully connected nodes adopt either status 1 or an undecided status. The proportion of time spent in these statuses is $1/1+1-a_1$ and $1-a_1/1+1-a_1$, respectively. The numerators in the expressions signify the transmission rate and dissipation rate of information at the particular node. As can be seen, when a node has a high degree, the rate of receiving a message from its many neighbors is 1, regardless of the actual

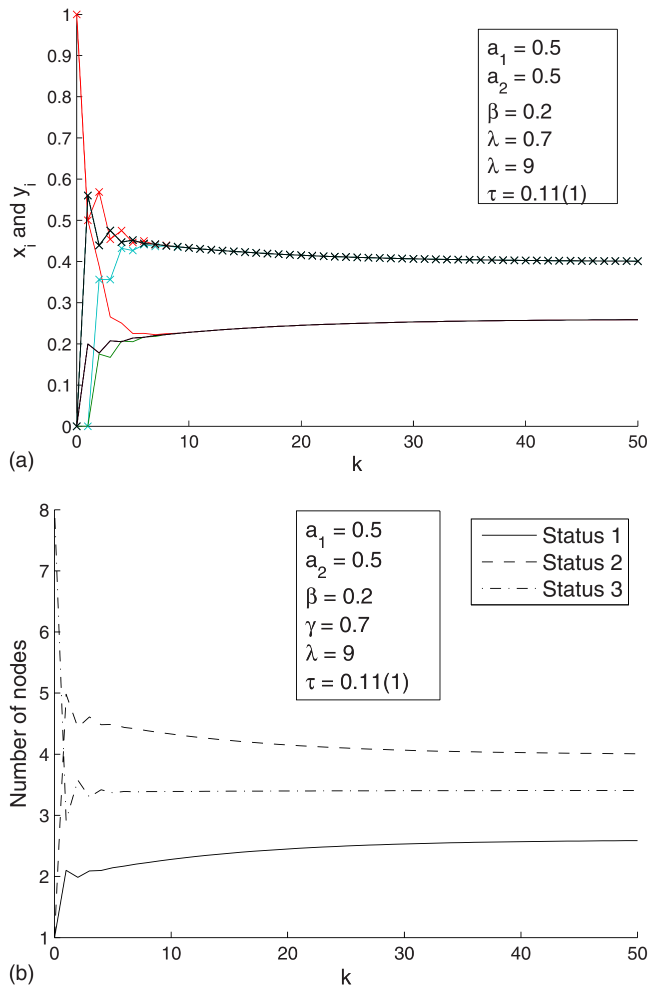


FIG. 6. (Color online) The evolution of (14) for a fully connected network with ten nodes. Both message types can pervade the network. (a) Evolution of x_i, depicted by solid lines and y_i, depicted by crossed lines. (b) The average number of nodes in each status.

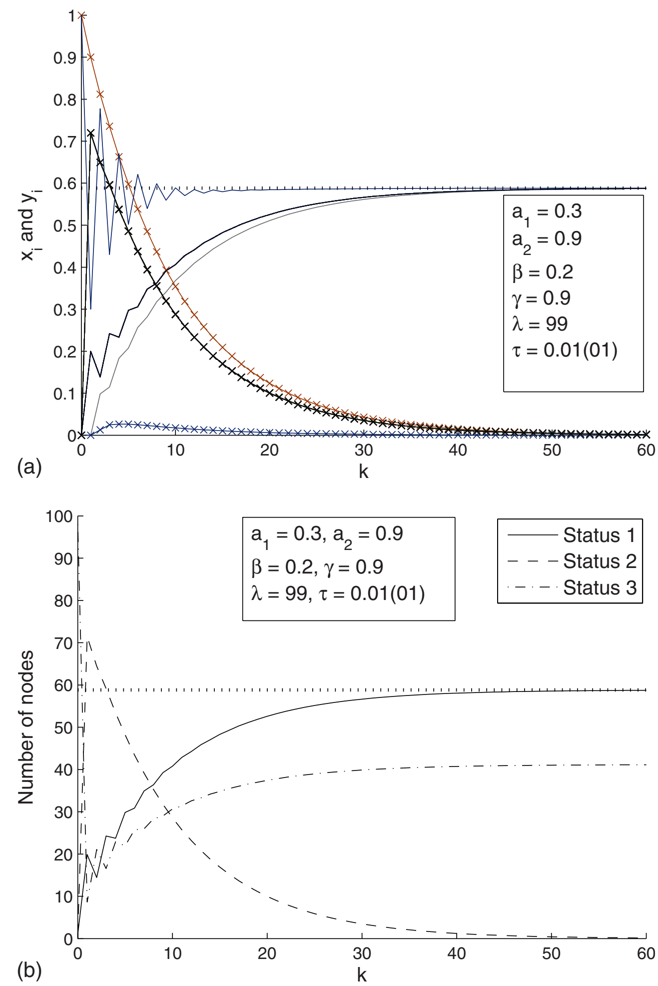


FIG. 7. (Color online) The model evolution for a 100-node fully connected network. (a) Evolution of x_i, depicted by solid lines and y_i, depicted by crossed lines. Dotted line is the approximating stable value for x_i given by Eq. (19). (b) The average number of nodes in each status. Dotted line is the stable number of nodes in status 1, as predicted by $N_1 = N\tilde{x} = 100/(2-a_1)$.

message transmission parameter. In fact, one can argue that this happens in an arbitrary topology as well. For a node i with high enough degree, the product

$$\prod_{j=1}^{N}[1-\beta a_{ij}x_j(k)]$$

in Eq. (4) tends to 0, i.e., $f_i \approx 1$ in the model equations, yielding the steady-state values as observed in the large star and fully connected graph. The degree value above which this behavior is observed depends on the values of the model parameters, as well as on the network topology. For example, consider Figs. 4 and 7. For the same model parameters, the fully connected graph requires much less nodes than the star in order to observe the stable values $\tilde{x} = \frac{1}{2-a_1}$ and $\tilde{y} = 0$. The fully connected topology allows the nodes to reinforce their decision more easily.

IV. BEHAVIOR OF THE MODEL ON COMPLEX NETWORK TOPOLOGIES

We now investigate the model behavior for complex network topologies. Of interest to us is the case when $\beta/1-a_1 > 1/\lambda$ and $\gamma/1-a_2 > 1/\lambda$ since theory and simulations both indicate that this is the only case when both rumor types are able to spread in the networks. All experiments start with one node in status 1 and one in status 2, which can change their status as time progresses. Moreover, in almost all experiments rumor 2 has relatively high parameter values in comparison to rumor 1, so as to observe its spread in the networks. Where applicable, we relate the results with those on regular networks.

A. Erdos-Renyi random networks

In this section we observe the model behavior on Erdos-Renyi (ER) networks. The model proposed by Erdos and Renyi described random graphs with N nodes in which every link exists with probability p. The degree distribution of these networks is Poisson, hence the homogeneous structure in the sense that all nodes have degree close to the average degree $\langle k \rangle = p(N-1)$. Also, in this model there is a critical probability value $p_c = \frac{1}{N}$ under which the resulting network

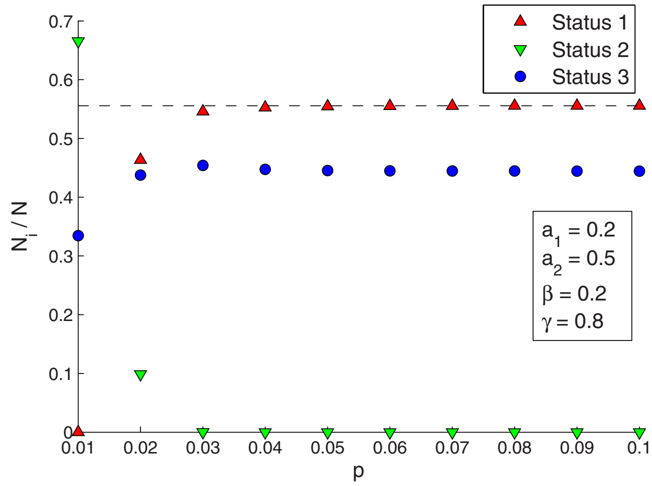


FIG. 8. (Color online) The steady-state behavior of the model for ER networks with $N=1000$ for different values of p, depicting the fraction of nodes in each status. For every value of p the results are the averages of 100 network realizations, each having a giant component with size of at least $0.9N$. The dashed line is the approximation of the number of nodes adopting rumor 1 given by $N_1=N\tilde{x}$, $\tilde{x}=\frac{1}{2-a_1}$.

consists of small disconnected components, and above which there is a giant component in the network containing $O(N)$ nodes. All the networks used in the simulations are generated with $p>p_c$, and the sources of the rumors are randomly placed in the giant component. Figure 8 shows the steady-state behavior of the model for ER networks with 1000 nodes for different values of p. Increasing p, the nodes have an increasingly higher degree, which, as can be seen, serves to promote rumor 1 at the expense of rumor 2, as could be expected from our previous discussion. The number of nodes in status 1 is accurately predicted by the approximation $N_1=N\tilde{x}$, $\tilde{x}=\frac{1}{2-a_1}$. Rumor 2 has chances to penetrate a random social network when the average degree of the nodes is low enough to allow it.

B. Small-world networks

In this section we investigate how the model behaves on small-world networks. It has been suggested that many real-world networks, and social networks among them have small-world characteristics. We use the Watts-Strogatz model as defined in [31] for generating the networks. ϕ denotes the rewiring parameter in the model, instead of β, to avoid confusion with the model parameter of this paper. The algorithm uses a starting ring lattice to construct a small-world network. In a ring lattice each node has $2K$ neighbors, K in the clockwise, and K in the anticlockwise direction. Each edge is rewired with probability ϕ, not allowing self-loops or multiple edges between nodes.

Figure 9 depicts the steady-state behavior of the model when the rewiring parameter ϕ is varied. The results are shown for networks of two different sizes for which $C(0)$; the clustering coefficient of the initial ring lattice is the same. The fraction of nodes in each status is given, as well as the clustering coefficient of each network, normalized by $C(0)$.

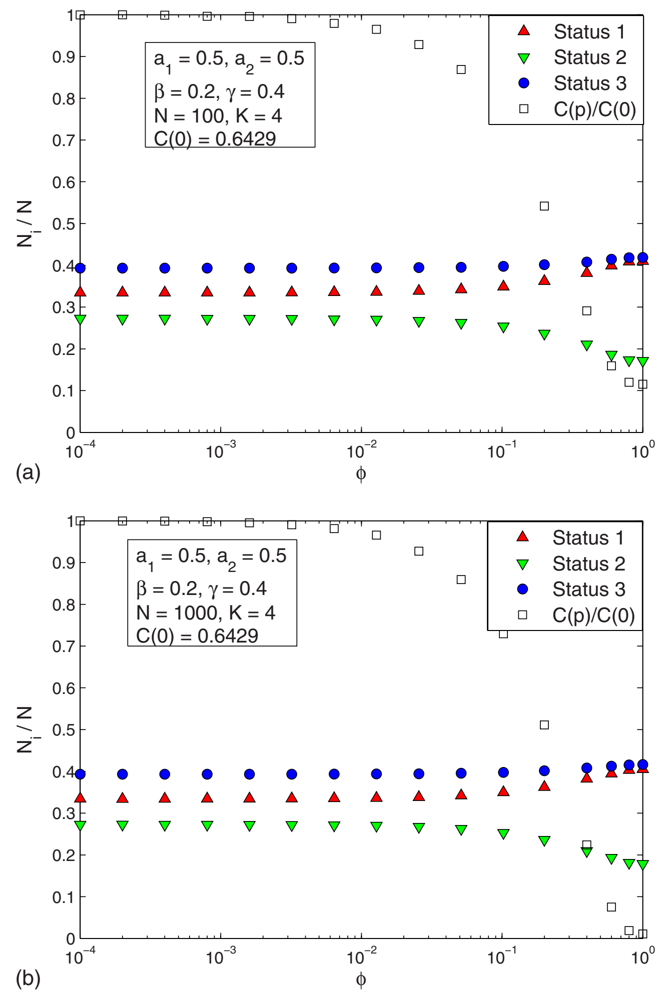


FIG. 9. (Color online) The steady-state behavior of the model for different values of ϕ. The fraction of nodes in each status is depicted, as well as the normalized clustering coefficient for the networks. (a) Results obtained by averaging over 50 network realizations for each ϕ, run for $k=400$ time units. (b) Results obtained by averaging over 25 network realizations run for $k=700$ time units.

Evidently, the results are the same due to the same level of clustering in the networks. Furthermore, for the particular values of the model parameters, status 2 is also present in the small-world networks. The cliquishness of the environment enables rumor 2 to occupy some of the nodes. However, as the networks become increasingly random, status 2 is diminished mostly at the expense of status 1. The lack of significant clustering in the random networks undermines the spreading of the second type of rumor.

Moreover, if the level of clustering in the networks is higher, i.e., the cliquish neighborhoods are larger, as in Fig. 10, then status 1 is the only remaining status, even if the parameters of rumor 2 are relatively higher than those of rumor 1. The high clustering of the networks acts to strengthen the influence of status 1, a result reminiscent of that in a fully connected graph. We further illustrate this by providing the time behavior of the model for a 1000-node ring lattice with $K=10$ (Fig. 11), and a small-world network generated from it with $\phi=0.01$ (Fig. 12), using the same parameters as in the previous experiment.

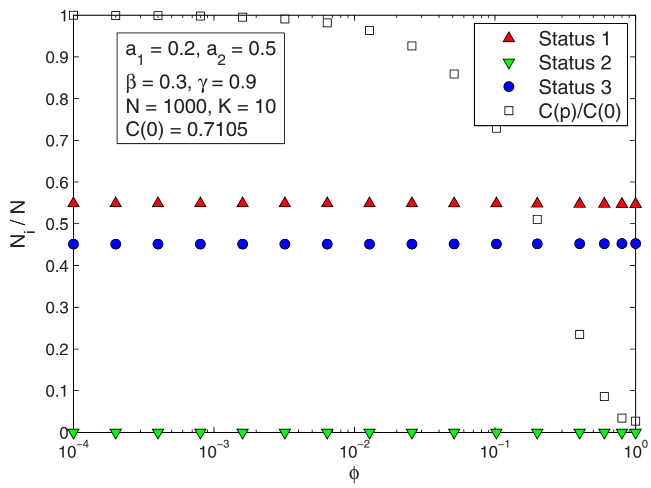
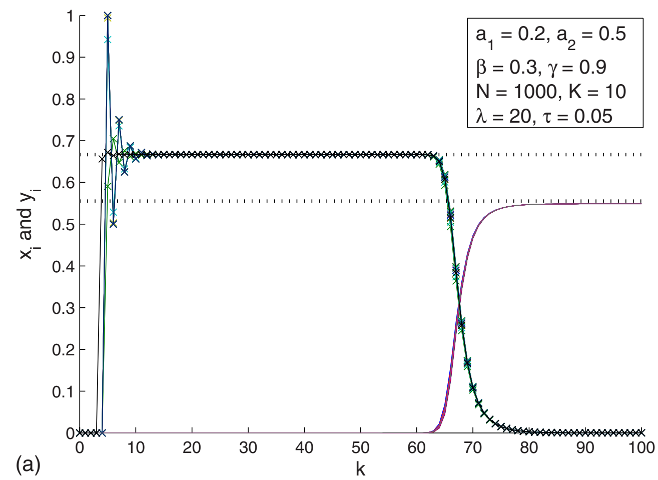


FIG. 10. (Color online) The steady-state behavior of the model for networks generated with the Watts-Strogatz algorithm, using a starting ring lattice with 1000 nodes and $K=10$.

The ring lattice is a highly clustered graph. Every node has a cliquish neighborhood, and specifically, in this example each 11 consecutive nodes on the ring are fully connected. Figure 11(a) shows the evolution of model (5) for nodes 1 to 11. Since in this case node 51 is in status 2 initially, in the beginning the group is influenced by the messages of rumor 2. The ring lattice has a long average path length. With the source of rumor 1 being node 501, it takes a longer time for the type 1 messages to traverse the network and reach the observed group. Once they do, however, rumor 1 suppresses the influence of rumor 2. Further, the values of x_i and y_i at the clique of 11 nodes are well approximated by Eq. (17) for a fully connected graph. Globally, Fig. 11(b) shows that both rumor types progressively affect nodes, with status 2 being slightly more successful in the beginning, due to its higher message transmission and remembrance parameters. In the long run, however, rumor 1 is greatly promoted due to the cliquishness of the environment, becoming the only rumor in the network.

We can now compare the behavior of the model on small-world networks with that on the ring lattice. Figure 12 shows a small-world network generated from the ring lattice in the previous example, with $\phi=0.01$. The initial conditions are the same as previously. The most obvious difference with the regular case is the speed of the dynamics. The process converges much more quickly on a small-world network due to the shortcut connections which greatly reduce the average distance between nodes. As a result, the spreading of the type 2 messages is greatly facilitated, and many nodes adopt rumor 2 in the beginning. However, there is a rapid spread in type 1 messages as well, and combined with the high clustering of this network, status 1 being preferable easily suppresses status 2. The end result is again reminiscent of that in fully connected graphs, with the stable x_i value of the observed nodes 1–11 being well approximated by $\tilde{x}=\frac{1}{2-a_1}$. Also, the total number of nodes in status 1 when the model stabilizes is well approximated by $N_1=N\tilde{x}$.

In conclusion, large neighborhoods in a small-world network which are clustered highly enough behave as fully connected graphs, strengthening the influence of the preferred rumor 1, and disabling the spread of rumor 2. Messages of type 1 are also easily spread through the long-range connections, quickly affecting the whole network. The type 2 messages could only be successfully spread if a small-world social network does not have large highly connected groups of nodes.



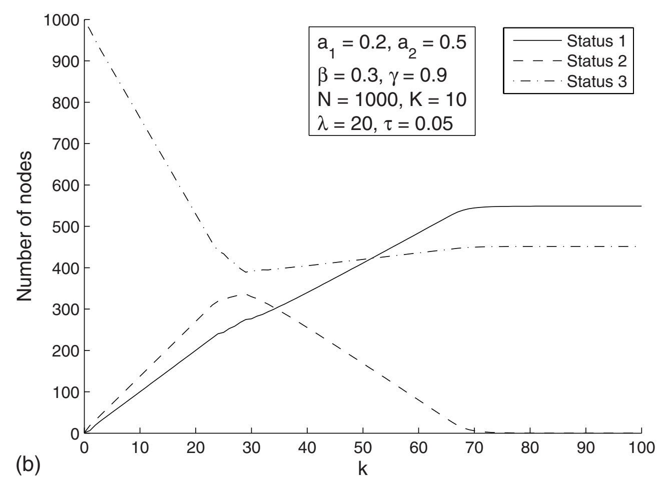


FIG. 11. (Color online) Behavior of the rumor spreading model (5) on a 1000-node ring lattice with $K=10$. Initially, node 51 is in status 2 and node 501 is in status 1. (a) The evolution of x_i and y_i for $i=1,2,\ldots,11$. Solid lines stand for x_i and crossed lines stand for y_i. Dotted lines are the approximating values of x_i and y_i for a fully connected network as given with Eq. (17). (b) The average number of nodes in each status.

C. Scale-free networks

In this section results of the model behavior on scale-free networks are presented. The original BA algorithm as given in [29] is used to construct the networks. One starts from a seed of m_0 connected nodes and adds a new node with $m \leq m_0$ links at each step according to the preferential attachment rule. This yields a network with average degree $\langle k \rangle = 2m$ and degree distribution $P_k \sim k^{-3}$.

Figure 13 shows the average number of nodes in each status as a fraction of N, as the parameter m of the BA

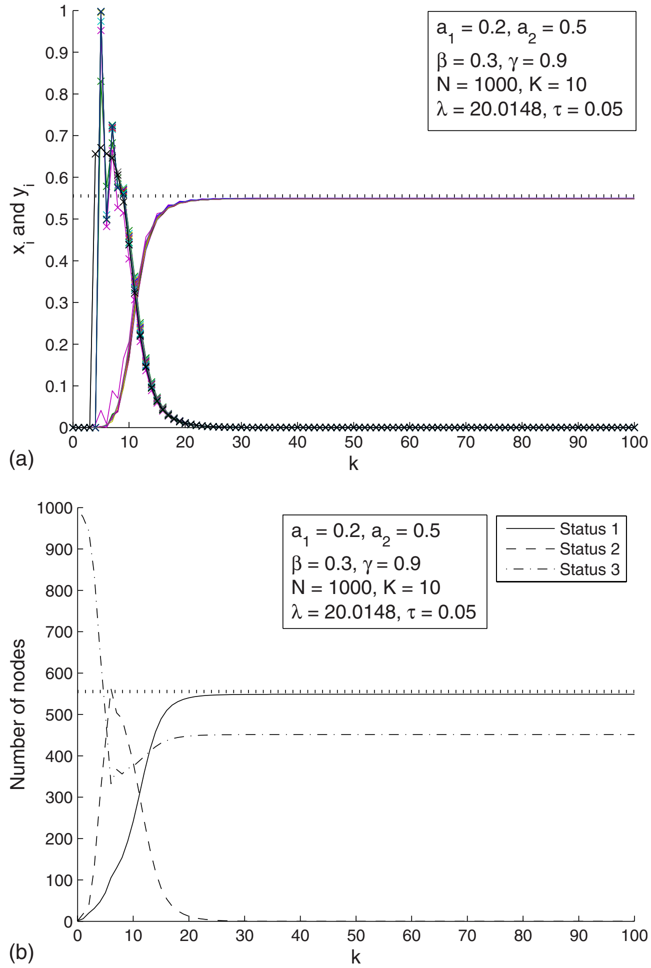


FIG. 12. (Color online) Behavior of rumor spreading model (5) on a small-world network with 1000 nodes generated from a ring lattice with $K=10$ using $\phi=0.01$. Initially, node 51 is in status 2 and node 501 is in status 1. (a) The evolution of x_i and y_i for $i=1,2,\ldots,11$. Solid lines stand for x_i and crossed lines stand for y_i. The dotted line is the stable value for x_i approximated by the expression in Eq. (17) for a fully connected network. (b) The average number of nodes in each status, obtained by summing the probabilities [Eq. (3)] for all nodes. Dotted line is the approximation $N_1=N\tilde{x}$.

algorithm is varied. The seed for the BA algorithm is a 20-node fully connected graph. For each value of m, the largest hub in the network has a degree high enough to act as a star, having $\tilde{x}=\frac{1}{2-a_1}$ and $\tilde{y}=0$. However, the total number of nodes in status 1 is different for different values of m in this particular setting. For $m=1$ and $m=2$ status 2 prevails, suggesting that the largest hub of the network does not have a far-reaching influence. Increasing m brings an increasing number of nodes in status 1, mostly at the expense of those in status 2, eventually expelling rumor 2 from the network for $m\geq 6$. As m is increased the algorithm creates a network with more hubs and higher degree nodes in general, the combination of which seems to promote the preferred rumor 1 heavily. One can imagine, as m increases, the number of nodes in status 1 to approach the dashed line.

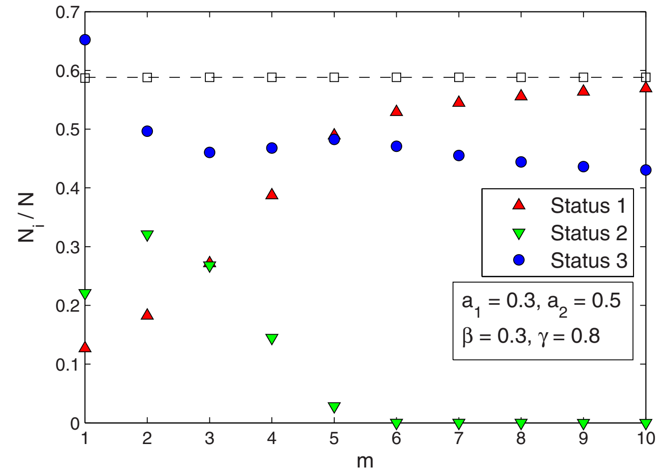


FIG. 13. (Color online) The average number of nodes in each status as a fraction of N, as m is varied. For each value of m, results are averaged over 100 network realizations, with the stable values taken at $k=100$. All networks are generated from a 20-node fully connected seed and have $N=1000$. The stable x value for the largest hub in the networks (squares) correspond to the approximation $\tilde{x}$ for the star in Eq. (13) (dashed line).

V. CONCLUSIONS AND DISCUSSION

In this paper we make an attempt to study how two different rumors, one of which is always considered first, propagate in a network. The model presented describes the interactions among nodes, and general results for the interplay of the two rumor types on different topologies are given. The key points of this paper are as follows;

(i) we suggest a model of rumor spreading over networks, in which each node can be in one of three possible states: 1, 2 and 3 corresponding to rumor 1 spreader, rumor 2 spreader and ignorant, respectively. The model is a natural generalization of the well-known epidemic SIS model, and reduces to the SIS model when some of the model parameters are zero. We confirm the existence of an intrinsic network threshold $\frac{1}{\lambda}$ for the spreading process to occur, as is already suggested by previous studies, where λ is the largest eigenvalue of the network's adjacency matrix A. The model has a unique stable fixed point, which implies irrelevance of the choice of initial rumor spreaders.

(ii) We find that the preferred rumor 1 is heavily promoted when the degree of nodes is high enough and/or when the network contains large clustered groups of nodes, expelling rumor 2. This is reflected in the model behavior on regular network topologies as well as on complex networks. Specifically, increasing the average degree in ER graphs yields these results, as does increasing the cliquish neighborhoods in small-world networks generated by the Watts-Strogatz model. In both cases the number of nodes adopting information type 1 is well approximated by $N\tilde{x}$, $\tilde{x}=\frac{1}{2-a_1}$. The behavior of the model is similar in BA networks as m is increased.

(iii) From the standpoint of the source of rumor 2, having to compete with a more reputable source is extremely challenging in many cases. Nevertheless, simulations show that it is possible for rumor 2 to occupy a nonzero fraction of the nodes in many cases as well. Specifically, in the Watts-Strogatz small-world model moderate clustering levels facilitate its adoption, while increasing randomness reduces it. For ER networks, a low average degree allows the coexistence of the two types of rumors. BA networks generated with a small m are also a pervading substrate for rumor 2.

Future research directions are numerous. Conditions in which status 2 has a "win situation" over status 1 remain to be determined. The stationary distribution of the Markov process for the network might be a promising tool for this matter. Nonergodic versions of the model, i.e., the ones when condition (7) does not hold, are also worth investigating. The problem of choosing the initial set of rumor spreaders which maximize the number of nodes in status 1 and 2 is natural in that setting. Finally, how well the model approximates reality is a key question to its usefulness. Hence, comparison with available real-world data is in order.

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